## Contents

Facilitator Information ..... i
Introduction ..... 1
I. (6 min) Hi-Lo Game ..... 2
II. (12 min) Player Strategies ..... 3
III. ( 10 min ) Worst \& Average Case Performance ..... 4
IV. (10 min) Effect of Input Size ..... 4
Applications ..... 5
Team Meeting Minutes (TMM): Rubric ..... 5
Facilitator - Answer Key ..... 1
Hi-Lo Game: Worksheet ..... 3
Hi-Lo Game: Graph. ..... 4

## Facilitator Information

## Learning Objectives (for content \& process)

After completing this activity, learners should be able to:

- Explain the pros \& cons of various simple search algorithms.
- Explain common tradeoffs between the complexity and performance of algorithms.
- Assess the performance of simple algorithms as a function of their input size N .

This activity should help learners develop teamwork and critical thinking skills.

## Prerequisites (for content \& process)

## Before starting this activity, learners should have:

- Previous experience with POGIL (useful but not necessary).
- Programming experience is not required for the activity, but is for some applications.


## Preparation

1. Print 1 activity per 2-3 students; put worksheet and graph on separate pages (not back to back)
2. Optional: Provide the worksheet on the board, a poster, or in presentation software, so teams can see each others' work easily.

## Activity Notes

- If students are unfamiliar with POGIL, it may be simpler to have teams of 3 (no Reflector).
- The facilitator should spend a minute or two introducing the activity.
- If time is limited
- introduce the game to the entire class
- skip: I.4, III.1, III.4, IV.
- While student teams work, the facilitator should circulate among the teams to monitor progress and help with problems, although the facilitator should avoid providing or confirming answers to any of the key questions.
- II. Player Strategies
- II.1. Report out: describe an algorithm. Summarize in table for class.
- II.3-4. Report out: rankings. Summarize in table for class.
- III. Worst \& Average Case
- III.3. Some teams may get stuck on random guessing.

Have everyone write down 1-2 random numbers, then list on board.
Notice: duplicates; min \& max; multiples of 5, 10; odd vs. even; etc.

- III.4. Report out: worst \& average case results. Summarize in table for class.


## Things to Do

- Review \& test
- ? Build histograms to better understand average \& worst case (see below)
- Part IV is too complex, at least for some CS1 students.
- Add more guidance on mean \& max counts
- Add sections on: implications for search: effort required to sort, tradeoffs.
- Add applications, resources, rubrics
- Add summary section.
- Add supporting resources.

| \# of guesses | possible values |
| :--- | :--- |
| 1 | 50 |
| 2 | 25,75 |
| 3 | $12,37,62,87$ |
| 4 | $6,18,31,43,56,68,81,93$ |

## Activity History

2010-06 drafted by Clif Kussmaul clif@kussmaul.org
2010-07...08 split into 2 activities (search and O() notation), revised
2011-01 piloted, revised
2012-12 revised
2013-08 revised based on experiences in India

## Introduction

start
time:

In computing, we often must search in a set for a particular item. As computer scientists, we are particularly interested in searching very large sets, with thousands or millions of values. For example, the Harvard University Library has roughly 16,000,000 volumes, and the US Library of Congress has roughly 22 million cataloged books, and over 100,000,000 total items. In this activity, we use a simple game to explore some basic searching algorithms. This will also help us explore more general concepts in algorithm design and analysis, so studying searching is useful even though very few of us may need to implement searching algorithms, since efficient techniques are part of most software libraries (APIs).

Before you start, complete the form below to assign a role to each member.
If you have 3 people, combine Speaker \& Reflector.

| Team | Date |
| :--- | :--- |

Reminders:

1. Note the time whenever your team starts a new section or question.
2. Write legibly \& neatly so that everyone can read \& understand your responses.

## I. (6 min) Hi-Lo Game

start
time:

Hi -Lo is a number guessing game with simple rules, played by school children.
a. There are two players - A and B.
b. Player A thinks of a number from 1 to 100 .
c. Player $B$ guesses a number.
d. Player A responds with
"too high", "too low", or "you win".
e. Players B and A continue to guess \& respond until B wins (or gives up).

1. (1 min) How many different answers can player A give? $\qquad$

2. (1 min) When does the game end?
3. (2 min) Play the game a few times to ensure that everyone understands the rules.
4. ( 2 min ) Optional: List up to 3 ways to clarify the rules.

## II. (12 min) Player Strategies

1. ( 3 min ) Identify $4-5$ different guessing strategies that Player B could use, and list them in the first column of the Hi-Lo Game: Worksheet.
Each strategy should describe a different approach to the game.
In computer science, we call such strategies algorithms. For example, one algorithm is:

Start at 1, and count up until the correct answer is found.

Try to have a mixture of simple and clever algorithms, including ones that young children could use. Before you continue, review progress with the facilitator.
2. ( 2 min ) Rank order the algorithms with regard to how quickly it will find the right answer. Write 1 for the fastest algorithm (fewest guesses) and 5 for the slowest algorithm (most guesses). Add the rankings to the worksheet in a column labeled Quick.
3. (2 min ) Rank order the algorithms with regard to how easy it is to describe or specify. (Suppose you had to explain each algorithm to a first-grader so that she could play the game.) Write 1 for the algorithm that is easiest to describe, and 5 for the algorithm that is hardest. Add the ranking to the worksheet in a column labeled Easy.
4. (1 min) For each algorithm, plot its Quick and Easy values on the Hi Lo Game: Graph.
5. (3 min) In complete sentences, describe the relationships between the Quick and Easy rankings, including what you see from the graph. Before you continue, review progress with the facilitator.

## III. (10 min) Worst \& Average Case Performance

1. ( 2 min ) To more accurately compare the speeds of these algorithms, we could have different people use different algorithms, and measure times with a stopwatch. Discuss and list the pros \& cons of this approach.
2. ( 3 min ) We could also compare the speed of algorithms using the number of guesses. For each algorithm, determine the worst case (maximum) number of guesses required to win. Add the numbers to the worksheet in a column labeled Worst.
3. ( 3 min ) For each algorithm, determine the average case (typical) number of guesses required to win. Add the numbers to the worksheet in a column labeled Average.
Note that the minimum number of guesses is always 1 - it's nice to be lucky.
4. (2 min ) List 3 reasons why the number of guesses could be a better way to compare algorithms. Before you continue, review progress with the facilitator.

## IV. (10 min) Effect of Input Size

## start

time:

1. (3 min) Assume that Player A chooses a number from 1 to 1000 (instead of 1 to 100 ). For each algorithm, what are the worst case \& average case number of guesses? Add the numbers to the worksheet in columns labeled " 1 K Worst" and " 1 K Average".
2. (4 min) Optional: Assume that Player A chooses a number from 1 to N .
(For example, $\mathrm{N}=100, \mathrm{~N}=1000, \mathrm{~N}=1,000,000$ )
For each algorithm, what are the worst case \& average case number of guesses in terms of N ? Add the expressions to the worksheet in columns labeled " N Worst" and " N Average".
(Hint: you've already done $\mathrm{N}=100$ and $\mathrm{N}=1000$; consider other values before generalizing to N .)
3. (3 min) Describe the pros \& cons of analyzing performance in terms of input size N .

## Applications

1. Team Meeting Minutes (TMM) - see common description (separate doc) and rubric (below).
2. Personal Reflection Memo (PRM) - see common description (separate doc).
3. Write a program (or set of programs) to play the game using each algorithm.

The program should count the guesses and print the total when the game ends.
Record how much time it takes to code each algorithm. Is this consistent with your "Easy" rating?
Play the game 5 times with each algorithm. Is the guess count consistent with your "Quick" rating?

## Team Meeting Minutes (TMM): Rubric

| SPECIFIC CRITERIA | RATING | COMMENTS |  |
| :--- | ---: | ---: | :---: |
| I: Comprehensively synthesizes <br> guessing algorithms. | $/ 4$ |  |  |
| II: Comprehensively synthesizes <br> rankings \& tradeoffs. | $/ 4$ |  |  |
| III: Comprehensively synthesizes <br> worst \& average case performance. | $/ 3$ |  |  |
| IV: Comprehensively synthesizes <br> N-analysis. | $/ 3$ |  |  |
| COMMON CRITERIA | RATING | COMMENTS |  |
| Begins with a summary of the activity; <br> ends with a summary of questions, <br> and a list of action items, if needed. | $/ 3$ |  |  |
| Mechanics \& format: <br>  <br> clearly provides all required information, <br> including team member roles \& timing data. | $/ 3$ |  |  |
| TOTAL |  |  |  |

## Facilitator - Answer Key

## I. (6 min) Hi-Lo Game

1. There are 3 possible answers - too high, too low, you win.
2. The game ends when B wins or gives up.

## II. (12 min) Player Strategies

1-4. See worksheet key below.
5. In general, simple algorithms tend to be slower, and faster algorithms tend to be more complex.

## III. (10 min) Worst \& Average Case Performance

1. Measuring time with a stopwatch is objective \& quantifiable, but may be affected by other factors math aptitude of the person playing the game, reaction time of the timer.
2-3. See worksheet key below.
2. Number of guesses is a better measure because it depends less on other factors.

## IV. (10 min) Effect of Input Size

1-2. See worksheet key below.
3. Pros \& cons of analyzing performance in terms of input size $N$.

Pros: generalizes from specific examples
Cons: more difficult calculations \& logic.

If teams have questions about random guessing, have each student write down 1-2 random numbers, then list or plot for entire class. Typical outcomes: no multiples of 10 or 5, more odds than evens

| 90 |  |  |  |  |  |  |  |  |  | $X$ |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 80 |  |  |  | X |  |  |  |  |  |  |
| 70 |  |  | X | X | X |  |  |  |  |  |
| 60 |  |  |  |  | X |  |  |  |  |  |
| 50 |  |  | X |  |  |  |  | X |  |  |
| 40 |  |  | XX | X |  | X |  | X |  |  |
| 30 |  |  |  |  |  |  | X |  |  |  |
| 20 |  |  | X |  |  |  |  | X |  |  |
| 10 |  | X | X |  | XX |  |  | X |  | X |
| 0 |  |  |  | XX |  |  |  | XX |  | XX |
|  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |


| \# | Player Algorithm |  | $\begin{aligned} & \stackrel{\rightharpoonup}{n} \\ & \stackrel{1}{\tilde{n}} \\ & \dot{N} \\ & = \end{aligned}$ |  |  | $\begin{aligned} & \stackrel{n}{\gtrless} \\ & \stackrel{n}{j} \\ & \equiv \end{aligned}$ |  |  |  | $\begin{aligned} & \dot{\#} \\ & \stackrel{y}{\gtrless} \\ & z \\ & \dot{~ M} \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | Guess numbers at random, ignoring "hi-lo" feedback, without memory (repeat guesses possible) | 5 | 2 | 10 | 1K | 50 | 10K | 500 | 10N | N/2 |
| B | Guess numbers at random, ignoring "hi-lo" feedback, but don't repeat guesses | 4 | 3 | 12 | 100 | 50 | 1000 | 500 | N | N/2 |
| C | Count up from 1 (or down from 100) | 3 | 1 | 3 | 100 | 50 | 1000 | 500 | N | N/2 |
| D | Count up by 10s, then down by 1 s (many variations) | 2 | 4 | 8 | 20 | 10 | $\begin{aligned} & 110 \\ & (30) \end{aligned}$ | $\begin{array}{r} 55 \\ (15) \end{array}$ |  |  |
| E | Guess 50, then 25 or 75, then 12 or 38 or 62 or 88 - keep track of min \& max possible values, divide range in half | 1 | 5 | 5 | 7 | 6 | 11 | 10 | $\log _{2} \mathrm{~N}$ | $\log _{2} \mathrm{~N}$ |

Hi-Lo Game: Worksheet

| \# | Algorithm Name | $\begin{aligned} & \frac{y}{3} \\ & 0 \end{aligned}$ | $\begin{aligned} & \lambda \\ & \stackrel{n}{\ddot{w}} \end{aligned}$ | $\pm$ 0 0 3 | $\begin{aligned} & 0 \\ & 0 \\ & \stackrel{0}{0} \\ & \frac{0}{0} \\ & \stackrel{2}{4} \end{aligned}$ | 눈 |  | $=\frac{\stackrel{*}{n}}{3}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | Count up from 1 until answer found |  |  |  |  |  |  |  |  |
| B |  |  |  |  |  |  |  |  |  |
| C |  |  |  |  |  |  |  |  |  |
| D |  |  |  |  |  |  |  |  |  |
| E |  |  |  |  |  |  |  |  |  |
| F |  |  |  |  |  |  |  |  |  |

## Hi-Lo Game: Graph


fewest guesses to most guesses
© 2014 Clif Kussmaul

